Review of Estimation: Key Terms

1. **Estimating the Mean of the Distribution:** Y has unknown mean , μ , and unknown variance, σ^2

2. **Random Sampling:** Sample n times…

   ex ante: $\{Y_i\} \sim Y \; iid \; , i = 1,...,n$; ***ex post***: $\{y_i\}$

3. **Estimators as Rules:** Estimators are random variables; the rule will generate different estimates depending on the particular sample

4. **(Point) Estimators v. estimates:**

   Estimator - $M(Y_1,...,Y_n) = \beta_0 + \beta_1 Y_1 + ... + \beta_n Y_n$ vs.

   Estimate - $M(y_1,...,y_n) = \hat{m} = \beta_0 + \beta_1 y_1 + ... + \beta_n y_n$ for the given sample

5. **Unbiased Estimator**: $E\left(M(Y_1,...,Y_n)\right) = \mu$ … on average, the rule gets it right

6. **Linear Unbiased Estimator (LUE)**: For estimating the mean of Y…

   $M = \beta_1 Y_1 + \beta_2 Y_2 + ... + \beta_n Y_n$, where $\sum_{i=1}^{n} \beta_i = 1$, so that $E(M) = \mu$.

7. **Best Linear Unbiased Estimator (BLUE)**:

   min $Var(M) = \sigma^2 \sum \beta_i^{\,2}$ s.t. $\sum_{i=1}^{n} \beta_i = 1$.

8. ***The Sample Mean is BLUE.*** (Does not depend on the distribution of Y.)

9. **Sample Statistics as Estimators:** Actual estimates will depend on the actual sample.

   a. ***Sample Mean*** (BLUE): $\bar{Y} = \frac{1}{n}\sum Y_i$, unbiased since $E(\bar{Y}) = \mu$

   b. ***Variance*** (unbiased): $S^2 = S_Y^2 = S_{YY} = \frac{1}{n-1}\sum (Y_i - \bar{Y})^2$, unbiased since

      $E(S_Y^2) = Var(Y) = \sigma^2$

   c. ***Standard Deviation*** (generally biased): $S_Y = \sqrt{S_{YY}}$

   d. ***Covariance*** (unbiased): $S_{XY} = \frac{1}{n-1}\sum (X_i - \bar{X})(Y_i - \bar{Y})$,

      $E(S_{XY}) = Cov(X, Y) = \sigma_{XY}$

   e. ***Correlation*** (generally biased): $\rho_{XY} = \frac{S_{XY}}{S_X S_Y}$

10. **Efficiency (of say, UEs):** Smaller variance, for any possible (true and unknown) parameter value

11. **Interval Estimator v. estimates:** Confidence Intervals are Interval Estimators;

    $[L(Y_1, ..., Y_n), U(Y_1, ..., Y_n)]$ vs. $[L(y_1, ..., y_n), U(y_1, ..., y_n)]$